Int. J. Heat Mass Transfer. Vol. 25, No. 1, pp. 149–150, 1982.
Printed in Great Britain

0017-9310/82/010149-02 $03.00/0

SHORTER COMMUNICATION

HEAT TRANSFER FROM MULTIDIMENSIONAL OBJECTS USING ONE-DIMENSIONAL SOLUTIONS FOR HEAT LOSS

L. S. LANGSTON
Mechanical Engineering Department, The University of Connecticut, Storrs, Connecticut, U.S.A.

(*Received* 30 *April* 1981 *and in revised form* 9 *July* 1981)

NOMENCLATURE

c_p,	specific heat;
h,	heat transfer coefficient;
k,	thermal conductivity;
L,	one-half thickness of body;
Q,	heat, energy;
T,	temperature;
u, v, w,	$(T - T_\infty)/(T_0 - T_\infty)$, non-dimensional temperature;
X, Y,	dimensional coordinates;
x, y,	coordinates, non-dimensionalized with L_1 and L_2, respectively;
α,	$k/\rho c_p$ = thermal diffusivity;
θ,	$\alpha\tau/L^2$ = non-dimensional time;
ρ,	density;
τ,	dimensional time.

Subscripts

0,	initial;
1,	body 1;
2,	body 2;
∞,	fluid.

INTRODUCTION

THE TEMPERATURE response charts, such as those given by Heisler [1] are useful for the rapid and accurate calculation of the temperature history of a heated or cooled body. Even today with computer programs available for solving almost any kind of heat conduction problem, the charts retain their usefulness. They can be found in most undergraduate heat transfer texts, such as in Kreith [2] and Holman [3], or in more extensive forms in Schneider [4]. Charts are usually given for the semi-infinite slab, the infinite plate, the infinite cylinder and the sphere, for constant temperature and convective boundary conditions.

Once the transient temperature distribution is known it is a relatively easy matter to calculate the total heat transfer from (to) the body. For the case of the infinite plate extending in the y-direction and $2L_1$ wide in the x-direction, this can be done by evaluating the heat transfer at the faces and integrating with respect to time, θ. Alternately, the total heat transfer can be found by calculating the stored internal energy of the plate at time, θ, and subtracting it from the original stored energy. The latter procedure yields

$$\frac{Q}{Q_0} = 1 - \int_0^1 u \, dx, \tag{1}$$

where u is the non-dimensional temperature distribution in the plate at time θ; Q is the total heat transferred from (to) the plate per unit area from $\theta = 0$ to $\theta = \theta$; $Q_0 = 2L_1\rho c_p (T_0 - T_\infty)$ is the initial energy per unit area of plate, relative to the fluid temperature, T_∞.

Values of Q/Q_0 were first presented in chart form by Gröber *et al.* [5] for simple shapes and can be found in standard heat transfer texts [2, 3]. Thus by using these charts, one can quickly answer the question: how long does one have to wait for a certain fraction of the internal energy of the body to be transferred to the fluid?

As was first shown by Berger [6] and Newman [7], solutions for the temperature distribution in 2- and 3-dim. shapes can be obtained from 1-dim. solutions by simple multiplication. For instance the temperature distribution in an infinite bar, $2L_1$ wide in the x-direction and $2L_2$ wide in the y-direction can be calculated by

$$w = uv \tag{2}$$

where $w = w(x, y, \theta)$ is the non-dimensional temperature at any point in the bar. In the product, uv, $u = u(x, \theta)$ is the solution for temperature in an infinite plate $2L_1$ wide and extending in the y-direction and $v = v(y, \theta)$ is the solution for an infinite plate $2L_2$ wide and extending in the x-direction. The intersection of the two plates form the infinite bar, with the point $x = 0$, $y = 0$ at the center of the bar.

The temperature of a 3-dim. rectangular body (e.g. a brick) could be calculated by intersecting three infinite plates and taking the product of the three temperatures. Use of temperature response charts for other shapes would yield the temperature history in finite cylinders, quarter infinite spaces, etc.

It is the purpose of this paper to show how the use of the heat transfer charts for a 1-dim. body can be extended to 2- and 3-dim. bodies, just as has been done for the case of the temperature distribution [i.e. equation (2)].

PROOF

The 2-dim. infinite bar, $2L_1$ by $2L_2$, will be used as an example. Following the same reasoning as was used to arrive at (1), the heat transferred from the rod in time θ can be written as

$$\frac{Q}{Q_0} = 1 - \int_0^1 \int_0^1 w \, dx \, dy \tag{3}$$

where x and y have been normalized by L_1 and L_2 respectively, and Q and Q_0 are defined as in (1), except they are each based on a unit length of the bar.

The temperature $w = w(x, y, \theta)$ is given by equation (2) so that (3) can be rewritten as

$$\frac{Q}{Q_0} = 1 - \int_0^1 u \, dx \int_0^1 v \, dy \tag{4}$$

since u is independent of y and v is independent of x. Combining (1) for both x and y with equation (4) yields an expression for the heat transfer from the bar in terms of 1-dim. chart solutions, and is given as

$$\frac{Q}{Q_0} = \frac{Q}{Q_0}\bigg|_1 + \frac{Q}{Q_0}\bigg|_2 - \frac{Q}{Q_0}\bigg|_1 \frac{Q}{Q_0}\bigg|_2. \tag{5}$$

Equation (5) for heat transfer is the counterpart of equation (2) for the 2-dim. temperature distribution.

An expression for Q/Q_0 for a 3-dim. body can be derived in the same way as was done to get (5). For a rectangular solid (e.g. a brick), $2L_1$ by $2L_2$ by $2L_3$, it can be shown that

$$\frac{Q}{Q_0} = \frac{Q}{Q_0}\bigg|_1 + \frac{Q}{Q_0}\bigg|_2\left(1 - \frac{Q}{Q_0}\bigg|_1\right) + \frac{Q}{Q_0}\bigg|_3\left(1 - \frac{Q}{Q_0}\bigg|_1\right)\left(1 - \frac{Q}{Q_0}\bigg|_2\right). \tag{6}$$

Of course equations (5) and (6) are also applicable to other shapes, such as a finite cylinder formed by the intersection of an infinite plate and an infinite cylinder. One needs only to use the appropriate 1-dim. chart to get values to put into equation (5) or (6).

EXAMPLE

As an example of the use of equation (5), consider a very long aluminum (k = 236 w/mC°, $\alpha = 97.5 \times 10^{-6}$ m²/s) rectangular bar that has the cross-sectional dimensions $2L_1$ = 0.2 m in the x-direction and $2L_2$ = 0.1 m in the y-direction. Initially the bar is at a uniform temperature T_0. Then at time $\tau = 0$, the bar is convectively cooled with a fluid at T_∞ and heat transfer coefficients h_1 = 236 w/m²C° at $X = \pm$ 0.1 m and h_2 = 94.4 w/m²C° at $Y = \pm$ 0.05 m. How long does it take for the bar to lose one-half of its internal energy relative to the fluid temperature?

From equation (5) it is seen that

$$\frac{Q}{Q_0}\bigg|_1 + \frac{Q}{Q_0}\bigg|_2 - \frac{Q}{Q_0}\bigg|_1 \frac{Q}{Q_0}\bigg|_2 = 0.5 \tag{7}$$

must be solved by trial-and-error. From equation (7) it is seen by inspection that the values of $(Q/Q_0)|_1$ and $(Q/Q_0)|_2$ will both be smaller that 0.5. Both of the 1-dim. bodies will cool more slowly (i.e. have a lower value of (Q/Q_0) for the same time τ) than the 2-dim. bar, since the latter has a higher surface-to-volume ratio.

By using the appropriate Biot numbers for each infinite plate and choosing a value of dimensional time τ (equal in both plates), values of $(Q/Q_0)|_1$ and $(Q/Q_0)|_2$ can be read from 1-dim. chart [2,3], until equation (7) is satisfied. By trial-and-error, this results in

$$\frac{Q}{Q_0}\bigg|_1 = 0.32, \quad \frac{Q}{Q_0}\bigg|_2 = 0.26,$$

yielding a cooling time of τ = 6.5 min for half of the internal energy of the bar to be transferred to the fluid. Had the question to be answered been one of calculating the heat transferred *given* a cooling time, the calculation would be straightforward, with no trial-and-error solution required.

CONCLUSIONS

The transient temperature distribution in multidimensional bodies can frequently be computed very simply by multiplying together appropriate 1-dim. transient solutions. In this paper this technique has been extended to include the calculation of heat transfer to or from a multidimensional body using 1-dim. solutions and either equation (5) or (6).

Acknowledgement—The author wishes to express his gratitude to W. W. Bowley for his suggestions on the subject of this paper.

REFERENCES

1. M. P. Heisler, Temperature charts for induction and constant-temperature heating, *Trans. ASME* **69,** 227–236 (1947).
2. Frank Kreith, *Principles of Heat Transfer*, 3rd edn., pp. 165–182. IEP (1973).
3. J. P. Holman, *Heat Transfer*, 5th edn., pp. 121–127. McGraw-Hill (1981).
4. P. J. Schneider, *Temperature Response Charts*. John Wiley (1963).
5. H. Gröber, S. Erk and Ulrich Grigull, *Fundamentals of Heat Transfer*, p. 52. McGraw-Hill (1961).
6. Franz Berger, Über die Berechnung des Temperaturverlaufes in einem Rechtkant beim Abkühlen und Erwärmen, *Z. angew. Math. Mech.* **8,** 479–488 (1928).
7. Albert B. Newman, Heating and cooling rectangular and cylindrical solids, *Ind. Engng Chem.* **28,** 545–548 (1936).